**Free-Response Section 1—with a calculator (30 points each)**

**1.** A ladder 16 feet long is leaning against the wall of a tall building. The base of the ladder is moving away from the wall at a rate of 1 foot per second.

**A.** How fast is the top of the ladder moving down the wall when the base of the ladder is 8 feet from the wall? (10 points)

**Answer:** The top of the ladder is moving down the wall at a rate of $\frac{1}{\sqrt{3}}$ feet/second.

Work:
let $x$=distance from wall to base of ladder
let $y$=distance from ground to top of ladder

$x^2 + y^2 = 256$ (1 point)

$2x \cdot x' + 2y \cdot y' = 0$ (3 points)

$x = 8, x' = 1, y = 8\sqrt{3}$ (3 points; 1 point for each value)

$2(8)(1) + 2(8\sqrt{3})y' = 0$

$y' = \frac{-1}{\sqrt{3}}$ feet/sec (3 points; 1 point for answer, 1 point for units, and 1 point for interpreting negative sign as meaning the ladder is moving *down* the wall; a decimal answer may be given of 0.577 ft/sec)

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**B.** Find the rate at which the angle $\theta$ between the ladder and the wall of the building is changing when the base of the ladder is 8 feet from the wall. (10 points)

**Answer:** The angle is increasing at a rate of $\frac{1}{8\sqrt{3}}$ radians/second.

Work:
$x = 16\sin\theta$ (2 points)

$\frac{dy}{dt} = 16\cos\theta\frac{d\theta}{dt}$ (2 points)

$1 = 16\cos\theta\frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{1}{16\cos\theta}$ from triangle $\cos\theta = \frac{\sqrt{3}}{2}$ (3 points)

$\frac{d\theta}{dt} = \frac{1}{16} \bullet \frac{2}{\sqrt{3}} = \frac{1}{8\sqrt{3}}$ radians/sec (3 points; 1 point for answer, 1 point for

units, and 1 point for interpreting positive sign as meaning the angle is increasing; a decimal answer may be given of 0.072 radians/sec; full credit may be awarded if student uses an incorrect value from part A, but the rest of the work is accurate)

Also possible to solve using the tangent function.

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**C.** A right triangle is formed by the ladder, the wall, and the ground. How fast is the area of this triangle changing when the base of the ladder is 8 feet from the wall? (10 points)

**Answer:** The area of the triangle is increasing at a rate $\frac{-4}{\sqrt{3}} + 4\sqrt{3}$ or $\frac{8}{\sqrt{3}}$ ft$^2$/sec.

Work:

$A = \frac{1}{2}xy$ (1 point)

$A' = \frac{1}{2}(x \bullet y' + y \bullet x')$ (3 points)

$x = 8,\ x' = 1,\ y = 8\sqrt{3},\ y' = \frac{-1}{\sqrt{3}}$

$A' = \frac{1}{2}\left(8 \bullet \frac{-1}{\sqrt{3}} + 8\sqrt{3} \bullet 1\right)$ (3 points for correct substitution)

$A' = \frac{-4}{\sqrt{3}} + 4\sqrt{3} = \frac{8}{\sqrt{3}}$ ft$^2$/sec. (3 points; 1 point for answer, 1 point for units, and 1 point for interpreting positive sign as meaning the area is increasing; a decimal answer may be given of 4.619 ft$^2$/sec; full credit may be awarded if student uses an incorrect value from part A or B, but the rest of the work is accurate)

**2.** A particle moves along the *x*-axis so that its position *s* in meters at any time $t \geq 0$ in seconds is given by the function $s(t) = 2t + 2\sin\left(\frac{\pi}{2}t\right)$.

**A.** Find the velocity function of the particle, and then determine the velocity at times $t=1$ and $t=2$. (10 points)

**Answer:**

$v(t) = s'(t) = 2 + \pi\cos\left(\frac{\pi}{2}t\right)$ (6 points)

$v(1) = 2$ m/s (2 points)

$v(2) = 2 - \pi \approx -1.1416$ m/s (2 points)

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**B.** Is the particle speeding up or slowing down at $t$=1.9 seconds? Justify your answer. (10 points)

**Answer:**
$a(1.9)<0$ and $v(1.9)<0$, therefore the particle is speeding up because $v$ and $a$ both have the same signs.

(5 points for answer; 5 points for valid justification)

**C.** Find the time $t$ on the interval [0, 3] at which the particle is farthest to the right. Justify your answer. (10 points)

**Answer:** The particle is farthest to the right at $t$=1.4393 seconds.

Work:
$v(t) = 0$
$2 + \pi \cos\left(\frac{\pi}{2} t\right) = 0$ (4 points)
$t$=1.4393 and $t$=2.5607 are critical numbers. (2 points)
$t$=1.4393 is a local maximum since position function goes from increasing to decreasing.
$s(1.4393)$=4.421
Check endpoints $s(0)$ =0 and $s(3)$ =4. (2 points)
$t$=1.4393 seconds is the time at which the particle is farthest to the right. (2 points)

**Free-Response Section 2—without a calculator**

**3.** Consider the curve given by $y^2 = 4xy + 1$.

**A.** Find $\frac{dy}{dx}$. (6 points)

**Answer:** $\frac{dy}{dx} = \frac{2y}{y - 2x}$

Work:
Uses implicit differentiation. (3 points)
$y^2 = 4xy + 1$
$2yy' = 4xy' + 4y$
$y' = \frac{2y}{y - 2x}$ (3 points)

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**B.** Find all points on the curve where the tangent line to the curve has slope 2. (8 points)

**Answer:** (0, 1) and (0, -1) (2 points; 1 point for each ordered pair)

Work:

$\frac{2y}{y-2x} = 2$ (3 points)

$2y = 2y - 4x$

$x = 0$ (1 point)

$y^2 = 4(0)y + 1$

$y^2 = 1$

$y = \pm 1$ (2 points; one point for each value)

**C.** Show that there are no points ($x$, $y$) on the curve where the line tangent to the curve is horizontal. (7 points)

**Answer:**

$\frac{2y}{y-2x} = 0$ (3 points)

$y = 0$ (1 point)

However, $(0) \neq 4x(0) + 1$ for any $x$, therefore the curve has no horizontal tangent. (3 points; other valid interpretations may be awarded full credit)

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**D.** Let *x* and *y* be functions of time *t* that are related by the equation $y^2 = 4xy + 1$. At time *t*=6, *y*=2 and $\frac{dy}{dt} = 7$. Find $\frac{dx}{dt}$ at time *t*=6. (9 points)

**Answer:** $\frac{dx}{dt} = \frac{35}{16}$

Work:

When $y = 2$, $(2)^2 = 4x(2) + 1$, so $x = \frac{3}{8}$. (3 points)

Using the chain rule and answer from Part A, $\frac{dy}{dt} = \frac{dy}{dx} \bullet \frac{dx}{dt} = \frac{2y}{y - 2x} \bullet \frac{dx}{dt}$

At $t = 6$,

$$7 = \frac{2(2)}{2 - 2\left(\frac{3}{8}\right)} \bullet \frac{dx}{dt}$$ (3 points)

$$7 = \frac{16}{5} \bullet \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{35}{16}$$ (3 points)

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**4.** The twice-differentiable function *f* is defined for all real numbers and satisfies the following conditions: $f(0) = -2$, $f'(0) = 3$, $f''(0) = -1$.

**A.** The function *g* is given by $g(x) = \tan(ax) + f(x)$ for all real numbers, where *a* is a constant. Find $g'(0)$ and $g''(0)$ in terms of *a*. (20 points)

**Answer:**
$g'(x) = f'(x) + a\sec^2(ax)$ (5 points)
$g'(0) = -1 + a = a - 1$ (5 points)

$g''(x) = f''(x) + 2a^2\sec^2(ax)\tan(ax)$ (5 points)
$g''(0) = -1$ (5 points)

**B.** The function *h* is given by $h(x) = \sin(kx) \bullet f(x)$ for all real numbers, where *k* is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of *h* at *x*=0. (10 points)

**Answer:** $y = -2kx$

Work:
$h'(x) = \sin(kx) \bullet f'(x) + f(x) \bullet k\cos(kx)$ (3 points)
$h'(0) = -2k$ (2 points)
$h(0) = \sin(0) \bullet f(0) = 0$ (2 points)
$y - 0 = -2k(x - 0)$ (1 point)
$y = -2kx$ (2 points)

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